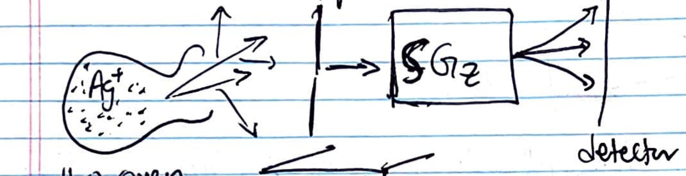
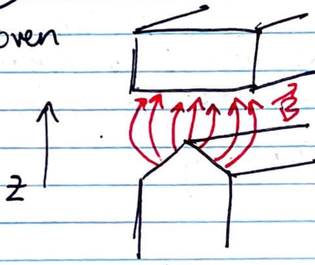
Week 7 Discussion Notes

§ Stern-Gerlach Exp





$\vec{B}$ inhomogeneous in z-direction.

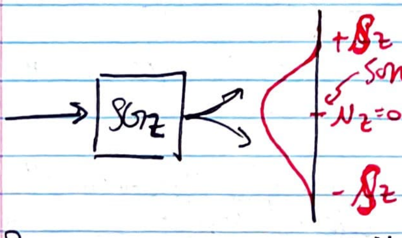
Silver atoms have a magnetic moment $\vec{\mu} = \frac{\vec{S}e}{m_e c}$ (similar to a spinning ball of charge, but not actually). In fact it is a relativistic effect.

Now, due to Ag electron configuration $\vec{\mu} \sim \vec{S}$ of a single electron. That is, what we will measure is the spin of the electron rather than the whole atom.

Recall (or don't) $F_z = \frac{\partial}{\partial z}(\vec{\mu} \cdot \vec{B}) \sim \mu_z \frac{\partial B_z}{\partial z}$

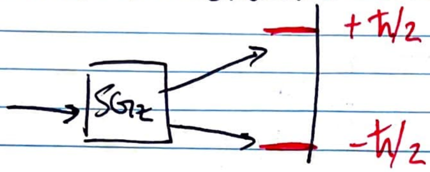
Now $\mu_0 > 0$, $S_z < 0 \Rightarrow$ downward force
$\mu_0 < 0$, $S_z > 0 \Rightarrow$ upward force.

What do we expect classically?

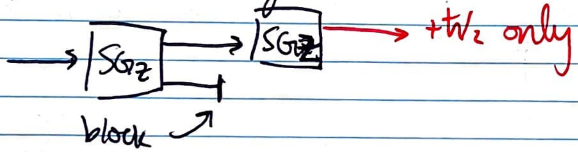


Because $\mu_z = \cos\theta\, |\vec{\mu}|$ (spherical coordinates !) and we assume θ to be random.

Instead we observe



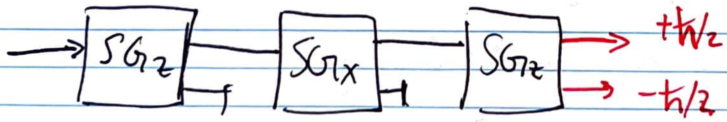
The conclusion, beyond electrons having spin, is that angular momentum is quantized. Even if we rotate to x or y direction, we get the same result. What about this



What should come out? How about

Classically, we expect $\vec{\mu} \sim \vec{z}$. Then $\vec{\mu} \cdot \vec{B} = 0$, for $\vec{B} \sim \vec{x}$, hence should see no splitting. But this isn't the case!

So far $SG_z \rightarrow SG_z$, we retain the knowledge of being $z+$. If we go $SG_z \rightarrow SG_x$, we "lose" that information. Now, what if we do:



We should expect only SG_z+ since we already picked that out earlier

No. We see splitting. We initially removed all S_z- but somehow the SG_x affected our atoms causing them to pick up a negative z-spin.

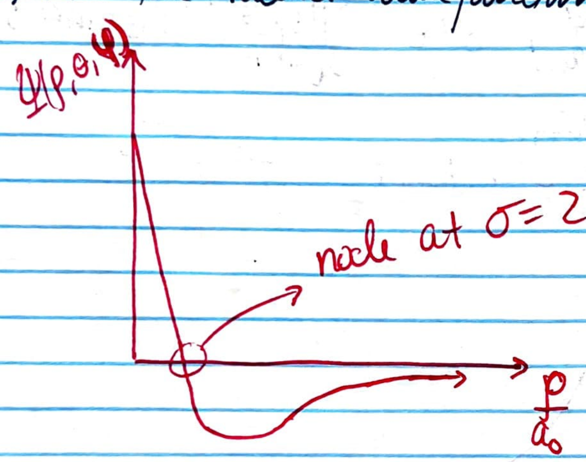
How??

Ex) Suppose we have the following wavefunction

$$\psi(\rho,\theta,\varphi) = \frac{1}{\sqrt{2\pi}} \left[\frac{Z}{2a_0}\right]^{3/2} (2-\sigma)\, e^{-\sigma/2}$$

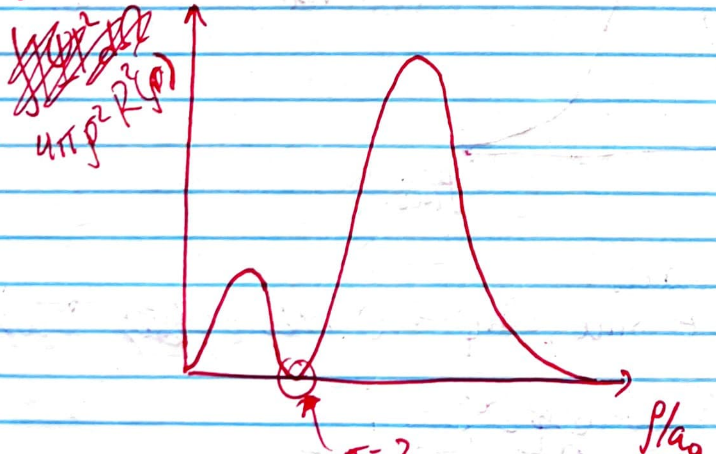
where a_0 is some constant and $\sigma = \frac{Z}{a_0}\rho$

a) Plot the radial wave function:



b) Plot the radial distribution function.

Really is $4\pi\rho^2 R^2(\rho)$, however, we can see that this is zero at $\rho = 0$ and will also be zero at $\sigma = 2$ (again).



c) Any angular nodes?

No? No dependence on θ or $\varphi \Rightarrow$ no angular node.

$\ell = 0$

d) What orbital is this?

$\ell = 0$ since no angular nodes. (s-type).
1 radial node.

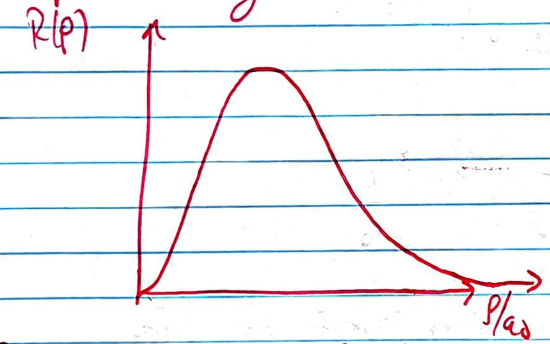
$\boxed{2s}$

Ex) Suppose we have the wavefunction

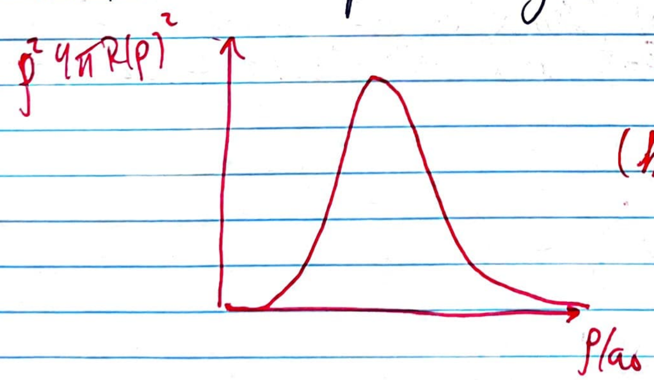
$$\psi(\rho,\theta,\varphi) = \left[\frac{1}{81\sqrt{15}}\left(\frac{2Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}\right]\left[\frac{1}{2}\sqrt{\frac{15}{\pi}}\cos\theta \sin\theta \sin\varphi\right]$$

a) Plot radial function.

Depends only on σ^2. Hence $\sigma^2 e^{-\sigma/3}$



b) Plot radial probability density.



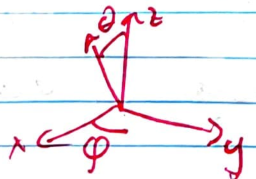
(basically the same).

c) Any angular nodes?

Yes! $\cos\theta \sin\theta \sin\varphi = 0$. ~~Note that~~ So when

~~$\cos\theta \sin\theta \sin\varphi$~~

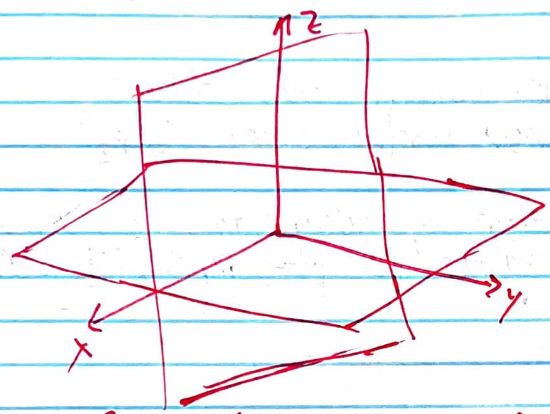
$\theta = \pm\frac{\pi}{2}$, $\theta = 0, \pi$ $\varphi = 0, \pi$.



So anywhere on z-axis, anywhere on x-axis
But note:

$\theta = \pm\pi$, φ can be anything : we have a node.
So entire x-y plane is a node?

Similarly. letting $\varphi = 0$: letting θ vary, we have
x-z plane is node?



So, 2 ~~node~~ angular nodes.

d) What orbital is this?
2 angular nodes $\Rightarrow$ d orbital.
No radial nodes $\Rightarrow$ 3d.

Furthermore, by investigating angular nodes, we have a $3d_{zy}$ orbital.

A note on why radial probability density is $4\pi\rho^2 R^2(\rho)$ vs ρ:

If you look at the surface area of a sphere of radius ρ, you will find it is $4\pi\rho^2$. So if we want the probability that the electron will be ρ away from the nucleus, we have to weight it by the surface area of this sphere.

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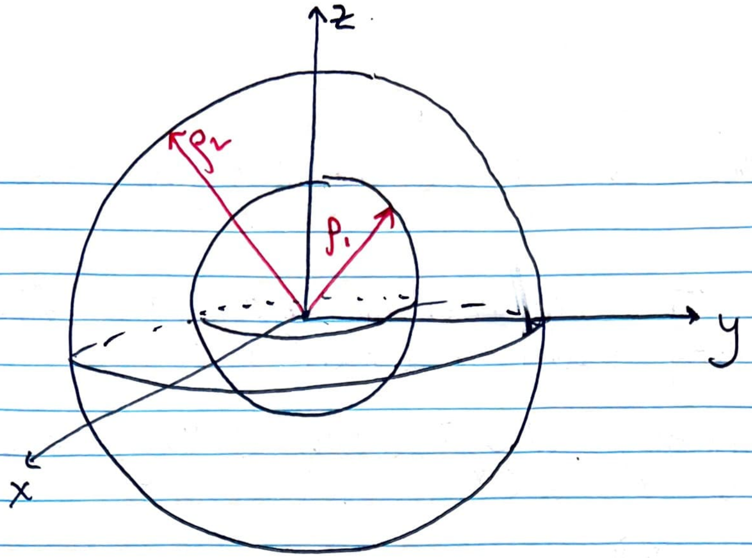

Inner sphere has surface area $4\pi\rho_1^2$
Outer sphere has surface area $4\pi\rho_2^2$

So to be a distance ρ_1 away from the nucleus, it has to be on the sphere. So our probability density is

$$4\pi\rho_1^2 R^2(\rho_1)$$

For the larger sphere ρ_2, we would have

$$4\pi\rho_2^2 R^2(\rho_2)$$

be its probability density.